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FOUR YEAR B.Sc. (CBCS) (Honours) DEGREE EXAMINATION,
APRIL/MAY 2025

FOURTH SEMESTER

Mathematics – Minor

Paper – 3 : RING THEORY

(w.e.f 2023-2024 Regulations)

Time : Three hours | Maximum : 70 marks

(No additional sheet will be supplied)

PART A — (5 × 4 = 20 marks)

Answer any FIVE questions

1. Let R be a Boolean ring and $a, b \in R$ then prove that

   (a) $a+a=0$

   (b) $a+b=0 \Rightarrow a=b$

2. Find zero divisors of the ring $(Z_{12}, +12, \times 12)$.

3. Define Sub ring and give an example.

4. Prove that field F has only ideals are $\{0\}$ and F.

5. Define principal Ideal.

6. Define a quotient ring.

7. Prove that every Homomorphic Image of a ring is a ring.

8. Define maximal ideal of a ring.

9. Find sum of the polynomials $f(x)=3+4x-2x^2$ and $g(x)-4-x+^2$ over the ring Z_5.

10. Prove that x^2+3x+2 is irreducible over Z_5.

PART B — (5 × 10 = 50 marks)

Answer ALL the following questions.

(Internal choice in each unit)

11. Show that every field is an Integral Domain.

Or

12. Show that $Z[i]=\{a+bi; a,b \in Z\}$ is an integral domain w.r. to addition and multiplication of complex numbers. Also show that it is not field.

13. Let R be a ring. Show that a nonempty subset S or R is a subring of R iff $a,b\in S\Rightarrow a-b\in S$, $a, b\in S$.

Or

14. If A and B are two ideals of a ring R then show that $A\cap B$ is also an ideal of R.

15. If R is a commutative ring with Unity and $a\in R$ then show that $Ra=\{ra: r\in R\}$ is an ideal of R.

Or

16. If A is an ideal of R and $x, y\in R$ then prove that (a) $x\in A\Leftrightarrow A+x=A$ (b) $A+x=A+y\Leftrightarrow x-y\in A$.

17. State and prove fundamental theorem of homomorphisms of rings.

Or

18. Show that an ideal M of a commutative ring R with Unity is maximal iff R/M is a field.

19. If R is an Integral'Domain then show that $R[X]$ is an integral Domain.

Or

20. State and prove Eisenstein Theorem on irreducible Polynomials.
